CS 533: Natural Language Processing **(Due: 03/31/20)**

Assignment 4

Instructor: Karl Stratos

- 3 problems: total 48 points (13 + 13 + 22)
- No collaboration
- Due by 11:59pm of the due date, no late submission accepted
- Use the provided LaTeX assignment template to write the answers. Upload the code as well.

Tip. For Problem 1 and 2, it will be easier to write a small program that computes all probabilties than to manually calculate the probabilities in question.

Problem 1: HMM

(1 + 6 + 6 = 13 points)

We consider a bigram HMM (o, t) that has emission probabilities $o(x|y)$ for all word types $x \in V$ and tag types $y \in \mathcal{Y}$ and transition probabilities $t(y'|y)$ for all $y \in \mathcal{Y} \cup \{*\}$ and $y \in \mathcal{Y} \cup \{\texttt{STOP}\}$ where $*$ and $\texttt{STOP}$ are special tags for the beginning and the end of a sequence. It defines the probability of $(x_1 \ldots x_n, y_1 \ldots y_n) \in V^n \times \mathcal{Y}^n$ by (letting $y_0 = *$)

$$p(x_1 \ldots x_n, y_1 \ldots y_n) = t(y_1|*) \times \left(\prod_{i=1}^{n} t(y_i|y_{i-1}) \times o(x_i|y_i)\right) \times t(\texttt{STOP}|y_n)$$

1. Assume the following data of tagged sequences $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)})$

   $$x^{(1)} = \texttt{the man saw the cut} \qquad y^{(1)} = \texttt{D N V D N}$$
   $$x^{(2)} = \texttt{the saw cut the man} \qquad y^{(2)} = \texttt{D N V D N}$$
   $$x^{(3)} = \texttt{the saw} \qquad y^{(3)} = \texttt{N N}$$

   Write all nonzero MLE parameter values of (o, t) estimated from this corpus. In particular, what are the nonzero emission probabilities for the word `cut` (i.e., possible tag types)?

2. Calculate the probability under the HMM that the third word is tagged with `V` conditioning on $x^{(2)}$ by running the forward and backward algorithms. That is, calculate

   $$\sum_{(y_1, y_2, y_3, y_4, y_5) \in \mathcal{Y}^5:\ y_3 = \texttt{V}} p(y_1, y_2, y_3, y_4, y_5 | \texttt{the saw cut the man})$$

   as a function of forward and backward probabilities. Round to 5 decimal places if necessary.

3. Similarly, calculate the probability that the fifth word is tagged with `N` conditioning on $x^{(1)}$:

   $$\sum_{(y_1, y_2, y_3, y_4, y_5) \in \mathcal{Y}^5:\ y_5 = \texttt{N}} p(y_1, y_2, y_3, y_4, y_5 | \texttt{the man saw the cut})$$

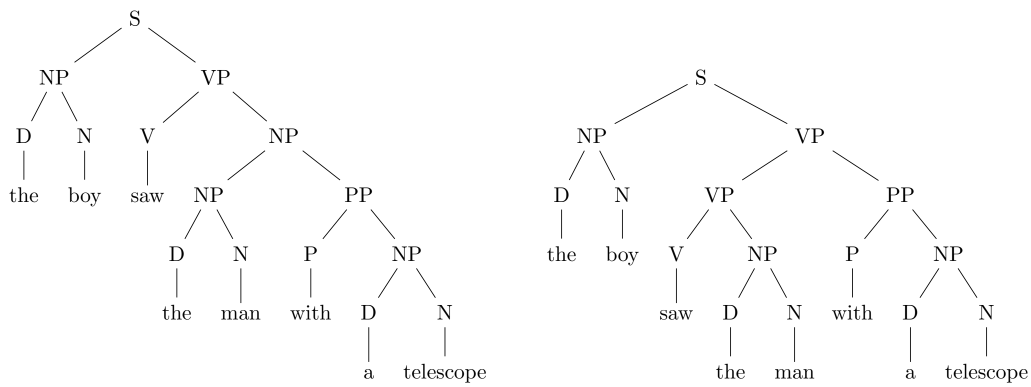
Problem 2: PCFG

(1 + 6 + 6 = 13 points)

We consider a PCFG in CNF (u, b) that has unary rule probabilities $u(X \to x|X)$ for all nonterminal types X and terminal types x and binary rule probabilities $b(X \to Y\ Z|X)$ for all nonterminal types X, Y, Z. It defines the probability of the parse tree as a product of the probabilities of the rules occurred in the tree. For instance, the first tree in the data described below is assigned the probability

$$\begin{aligned}
&u(\text{D} \to \text{the}|\text{D})^2 \times u(\text{D} \to \text{a}|\text{D}) \times u(\text{N} \to \text{boy}|\text{N}) \times u(\text{N} \to \text{man}|\text{N}) \times u(\text{N} \to \text{telescope}|\text{N}) \times u(\text{V} \to \text{saw}|\text{V}) \\
&\times u(\text{P} \to \text{with}|\text{P}) \times b(\text{S} \to \text{NP VP}|\text{S}) \times b(\text{NP} \to \text{D N}|\text{NP})^3 \times b(\text{NP} \to \text{NP PP}|\text{NP}) \times b(\text{PP} \to \text{P NP}|\text{PP}) \\
&\times b(\text{VP} \to \text{V NP}|\text{VP})
\end{aligned}$$

1. Assume the following data of 2 parses for sentence $x =$ (`the boy saw the man with a telescope`):



   Write all nonzero MLE parameter values of (u, b) estimated from this corpus.

2. Calculate the probability under the PCFG that NP spans $(4, 8)$ (i.e., "the man with a telescope") conditioning on x by running the inside and outside algorithms. That is, calculate

$$\sum_{\tau \in \mathbf{GEN}(x):\ \text{root}(\tau,4,8)=\text{NP}} p(\tau|x)$$

   (where $\mathbf{GEN}(x)$ is the set of all possible parses and $\text{root}(\tau, i, j)$ is the nonterminal at the root of a subtree spanning (i, j)) as a function of inside and outside probabilities.

3. Similarly, calculate the probability that VP spans $(3, 5)$ (i.e., "saw the man") conditioning on x

$$\sum_{\tau \in \mathbf{GEN}(x):\ \text{root}(\tau,3,5)=\text{VP}} p(\tau|x)$$

Problem 3: Programming (CRF) (1 + 1 + 1 + 1 + 1 + 1 + 8 + 8 = 22 points)

You will implement the CRF inference layer in a tagger based on bi-directional LSTMs (BiLSTMs).[1] As usual, download the starter kit provided and follow the instructions below.

> As a warmup, run `python main.py /tmp/pos data/ptb-wsj-snippet --train --loss greedy --nochar`. This trains a BiLSTM greedy tagger without character-level information on a tiny POS tagging dataset in which train/validation/test portions are the same. (When the word labels do not have the BIO format, the code automatically sets the performance metric to be per-position accuracy.) Also run `python main.py /tmp/pos data/conll2003-snippet --train --loss greedy --nochar`. This trains the tagger on a tiny NER tagging dataset in which train/validation/test portions are the same. (When the word labels have the BIO format, the code automatically sets the performance metric to be global F_1.)

[1] Neural Architectures for Named Entity Recognition (Lample et al., 2016)

1. Take a look at tagged sequences in `ptb-wsj-snippet/` and `conll2003-snippet/` and the class `TaggingDataset` and explain in your own words how the program organizes labeled sequences into batches. In particular: (1) Why do we sort sequences by length? (2) When you set batch size N, does that mean every batch will contain N sequences? (3) Is there any padding at the word sequence level? (4) How are characters organized? (5) Is there any padding at the character sequence level?

2. Take a look at `evaluate` method in `BiLSTMTagger` and explain how `output['acc']` and `output['f1_<all>']` are computed from ground-truth and predicted labeled sequences.

3. The greedy tagging loss is implemented for you in `crf.py`. Let $(x^{(1)}, y^{(1)}) \dots (x^{(N)}, y^{(N)})$ denote N labeled sentences in a batch where $x^{(i)} \in V^T$ and $y^{(i)} \in \{1 \dots L\}^T$. For $i = 1 \dots N$, let $h^{(i)} \in \mathbb{R}^{T \times L}$ denote the output of the BiLSTM layer given input $x^{(i)} \in V^T$ representing label scores at T positions (i.e., `scores`[i]). Give a precise formula for the greedy loss computed on this batch as a function of $x^{(i)}, y^{(i)}, h^{(i)}$.

4. You can make the tagger a function of characters by simply removing `--nochar` in the command. Explain how character-level information is incorporated. In particular: (1) Given a word $w \in V$, how are its characters $c_1 \dots c_{|w|} \in \mathcal{C}$ ($\mathcal{C}$ is the set of all character types) used to produce an embedding? (2) What is the final dimension of a word representation to be fed into the BiLSTM layer which produces the label scores $h \in \mathbb{R}^{T \times L}$ (as a function of `wdim` and `cdim`)?

5. Take a look at `CRFLoss` in `crf.py`. (1) What are the parameters of this layer? (2) Give a precise formula for the single scalar **score**(h, y) computed by the layer in terms of the CRF parameters where $h \in \mathbb{R}^{T \times L}$ is label scores for an input sequence and $y \in \{1 \dots L\}^T$ is a specific label sequence. This is implemented in `score_targets`. (3) What is the loss (computed in `forward`) as a function of the label sequence scores in (2)?

6. What quantities are `compute_normalizers_brute` and `decode_brute` computing and how? What is the computational complexity of these methods in terms of label set size L and sequence length T?

7. **Forward algorithm.** Implement `compute_normalizers` with complexity $O(|L|^2 T)$ to compute the same quantity computed by `compute_normalizers_brute`. The tensor `prev` $\in \mathbb{R}^{B \times L}$ contains the active part of the conceptual $B \times T \times L$ dynamic programming table, that is

$$\texttt{prev}[i][y] = \log\left(\sum_{y_1 \cdots y_{t-1}} \exp\left(\mathbf{score}(x_1^{(i)} \dots x_t^{(i)}, y_1 \dots y_{t-1} y)\right)\right)$$

where $t = 1 \dots T$ is implicitly traversed (since we only need the information from $t-1$ to get t). You will have to be careful with batch dimensions using commands like `unsqueeze` and `expand`. You will have to be careful with details like the directionality of `self.T`: its value at index (y', y) means score for $y \to y'$ not $y' \to y$. Do not introduce any for loops except for the one iterating over positions $1 \dots T$. Use `logsumexp`. **You have to pass the test `test_compute_normalizer` in `test_crf.py` to get any credits for this problem (run `python test_crf.py`).** (In the reference implementation, each line is a one-liner.)

8. **Viterbi algorithm.** Implement `decode` with complexity $O(|L|^2 T)$ to compute the same quantity computed by `decode_brute`. The tensor `prev` $\in \mathbb{R}^{B \times L}$ contains the active part of the conceptual $B \times T \times L$ dynamic programming table, that is

$$\texttt{prev}[i][y] = \max_{y_1 \cdots y_{t-1}} \mathbf{score}(x_1^{(i)} \dots x_t^{(i)}, y_1 \dots y_{t-1} y)$$

where $t = 1 \dots T$ is implicitly traversed (since we only need the information from $t-1$ to get t). PyTorch's `max` operation will yield indices, which you will store in the list `back` for backtracking purposes. After the main loop, you will backtrack from the final maximizers through `back` and store results into `tape` $\in \mathcal{Y}^{B \times T}$ (whose order will be reversed at return). Again be careful with dimensions/details and do not introduce any for loops. **You have to pass the test `test_decode` in `test_crf.py` to get any credits for this problem (run `python test_crf.py`).** (In the reference implementation, each line is a one-liner.)